TEMPERATURE DEPENDENCES OF FREE SURFACE EQUILIBRIUM CONDITIONS FOR A MAGNETIC FLUID IN AN ANNULAR GAP

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UDC 537.63

An isothermal problem on the equilibrium of a free magnetic fluid surface in an annular gap is considered. Conditions are investigated for the existence of a solution for different system temperatures.

The question of the existence of axisymmetric equilibrium modes of a free magnetic fluid surface (MF) in the annular gap between two coaxial cylinders (Fig. 1) was studied in [1]. A current I flows in the inner cylinder (radius r_2), while the outer cylinder is nonconducting. The temperature T (everywhere within) of the MF was assumed to be homogeneous and constant, equal to the temperature of the solid walls, and the properties of the magnetic medium were constant independent of T. Within the framework of such an isothermal formulation, existence conditions are found for equilibrium modes, and their properties are investigated.

It is interesting to examine the equilibrium conditions for the configuration displayed in Fig. 1 with the temperature dependences of the MF properties taken into account while remaining here within the framework of the isothermal formualtion. As in [1], we shall here consider the case of the MF being weightless.

1. Temperature Dependences of the MF Properties. Taking into account that the MF magnetization can reach a high level in the saturation state without developing hysteresis effects in the zeroth magnetic field, magnetic fluids should be referred to the category of superparamagnets. According to [2], the magnetic susceptibility of superparameters χ can be represented in the following form as a function of the temperature T:

$$\chi(T) = \frac{nV^2(T)}{3k_B T} I_s^2(T), \tag{1}$$

where $k_B = 1.38 \cdot 10^{-23}$ J/K.

If we take a linear thermal-expansion law, then $V(T) = V_0(1 + \alpha_V T)$.

The temperature T does not exceed the Curie temperature θ, which is the point of a phase transition of the second kind. Here $\theta \sim 10^3$ °K, while the coefficient of volume expansion varies within the limits $\alpha_V = 10^{-5}$–10^{-6} °K^{-1}. Consequently, the thermal expansion can be neglected to a high degree of accuracy by setting

$$V(T) = V_0. \tag{2}$$

The function $I_s(T)$ has the form [2]

$$I_s(T) = \frac{m_*}{\sqrt{3}} \left[\frac{3}{\theta}(\theta - T)\right]^{1/2}, \tag{3}$$

where $m_* = I_s(0)$ is a certain constant.

Using the notation $m_0 = m_* V_0$, we obtain from (1)-(3)

$$\chi(T) = \frac{nm_0^2}{3k_B\theta} \frac{\theta - T}{T}. \tag{4}$$

The quantity m_0 in (4) is the magnetic moment of a ferromagnet particle at T = 0.

The temperature dependence of the surface tension coefficient σ has the following form for the majority of fluids [3]

Institute of High Temperatures, Academy of Sciences of the USSR, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 48, No. 5, pp. 843-847, May, 1985. Original article submitted February 27, 1984.

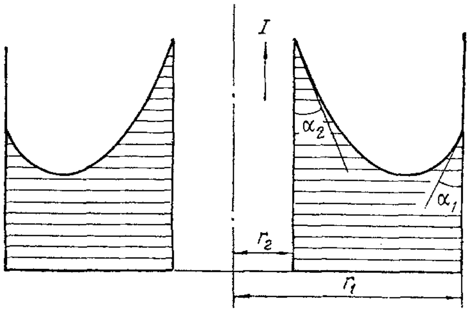

Fig. 1

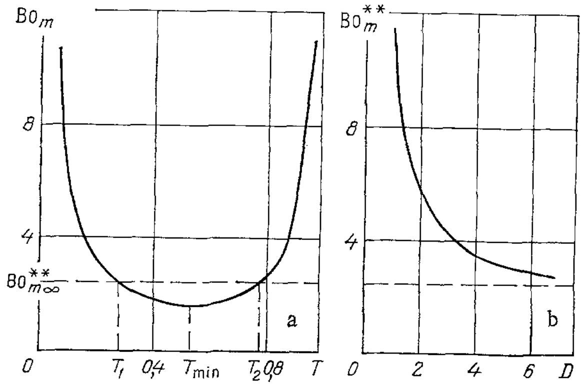

Fig. 2

Fig. 1. System configuration.

Fig. 2. a) Dependence of the Bond magnetic number (Bo_m) on the temperature (T) (liquid MĖ-22, I = 0.3 A); b) critical values of the Bond magnetic number (Bo_m^{**}) on the gap width (D) (wetting angle is α = 60°).

$$\sigma(T) = \gamma (T_c - T)^p, \tag{5}$$

where $\gamma > 0$ is a constant; p = 1.23, and T_c is the temperature of the critical Van der Waal's isotherm.

It is convenient to seek the solution of the equilibrium problem in dimensionless form. Then the Bond magnetic number Bo_m will be the characteristic parameter that equals in this problem [1]

$$Bo_m = \frac{\mu_0 \chi I^2}{8\pi^2 \sigma r_2},$$

where $\mu_0 = 4\pi \cdot 10^{-7}$ Gn/m.

Taking (4), (5) into account, we obtain

$$Bo_m = \frac{\mu_0 n m_0^2 I^2}{24\pi^2 \gamma k_B \theta r_2} \, \frac{\theta - T}{T (T_c - T)^p}. \tag{6}$$

We go over to the dimensionless temperature $\bar{T}$ by setting

$$T = T_c \bar{T}. \tag{7}$$

It is hence seen that $0 < \bar{T} < 1$. Taking (6) and (7) into account, the expression for Bo_m can be written in the form

$$Bo_m(\bar{T}) = C I^2 \frac{Q - \bar{T}}{\bar{T}(1 - \bar{T})^p}, \tag{8}$$

$$C = \frac{\mu_0 n m_0^2}{24\pi^2 \gamma k_B \theta r_2 T_c^p}, \tag{9}$$

where $Q = \theta / T_c$. Furthermore, we omit the bar over the $\bar{T}$ everywhere. The current I is liberated separately in (8) since the change in Bo_m (in experiment) occurs for the very same temperature.

2. Investigation of the Equilibrium Conditions. Let us introduce the dimensionless wid width of the gap $D = r_1/r_2$, and let the wetting angles on the cylinder walls be identical: $\alpha_1 = \alpha_2 = \alpha$.

It was shown in [1] that equilibrium modes exist if and only if $Bo_m < Bo_m^{**}$, where

$$Bo_m^{**} = \frac{4.9 + 5 \sin \alpha}{(D - 1)^{1,38}} + 1,1\alpha + 1. \tag{10}$$

We select the case of "low" and "high" currents separately to study the influence of the temperature dependences $\chi(T)$ and $\sigma(T)$ on the existence of equilibrium modes.

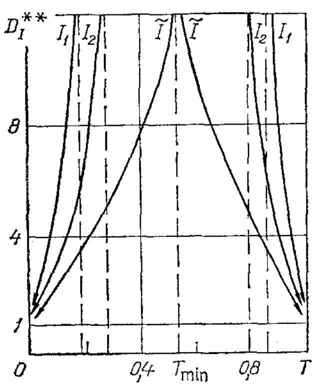

Fig. 3

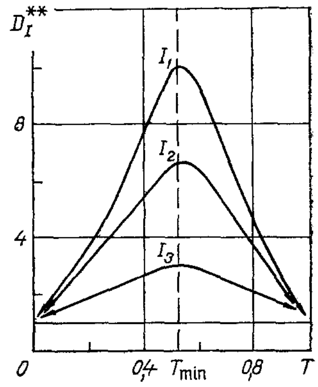

Fig. 4

Fig. 3. Dependence of the critical gap width (D_I^{**}) on the temperature (T) in the case of "low" currents (fluid MÉ-22; I_1 = 0.25 A; I_2 = 0.3 A; $\tilde{I}$ = 0.37 A). Dashed lines are asymptotes.

Fig. 4. Dependence of the critical gap width (D_I^{**}) on on the temperature (T) in the case of "high" currents (fluid MÉ-22; I_1 = 0.39 A; I_2 = 0.45 A; I_3 = 0.6 A).

"Low" Currents. Let $0 < I < \tilde{I}$ (for I = 0 the equilibrium modes always exist). Here $\tilde{I}$ is that value of the current for which $Bo_m(T) = Bo_{m\infty}^{**} = 1.1\alpha + 1$.

It follows from a comparison of (8), (9), and (10) that

$$\tilde{I} = (1 - T_{\min})^{p/2}\left[\frac{(1.1\alpha + 1)\,T_{\min}}{C\,(Q - T_{\min})}\right]^{1/2}, \tag{11}$$

where T_{min} is the temperature corresponding to the minimum of the function $Bo_m(T)$ (Fig. 2)

$$T_{\min} = \frac{Q\,(p+1) - \sqrt{Q^2(p+1)^2 - 4Qp}}{2p}. \tag{12}$$

Comparing (8) and (10), the deduction can be made that a temperature band exists $[T_1(I), T_2(I)]$ that contains the point T_{min} at which equilibrium exists for any values of the gap width D. Here $T_i(I)$, i = 1, 2, are roots of the equation

$$CI^2\frac{Q - T}{T\,(1 - T)^p} = 1.1\alpha + 1. \tag{13}$$

The transcendental algebraic equation (13) for the unknown T has two roots, as follows from the figure: $0 < T_1 < T_{min}$, $T_{min} < T_2 < 1$. The values of T_1 and T_2 can be found by a numerical method.

As I grows, the temperature band $[T_1(I), T_2(I)]$ diminishes, going over into the point T_{min} for $I = \tilde{I}$.

If $T \in A = \{[0,\ T_1(I)] \cup [T_2(I),\ 1]\}$, then a domain can be indicates for the existence of equilibrium modes in the parameter D, in other words, for each value of $T \in A$ equilibrium modes exist if $D < D_I^{**}(T)$, and do not exist otherwise.

The value of $D_I^{**}(T)$ is determined from (8) and (10):

$$D_I^{**}(T) = \left[\frac{4.9 + 5\sin\alpha}{CI^2\dfrac{Q - T}{T\,(1 - T)^p} - 1.1\alpha - 1}\right]^{0.73} + 1. \tag{14}$$

Graphs of the function $D_I^{**}(T)$ are shown in Fig. 3 for different values of I.

"High" Currents. Let us consider the case $I \geqslant \tilde{I}$, where $\tilde{I}$ is given by (11), as before. In this case $D_I^{**}(T)$ is defined for any T [in conformity with (14)], i.e., for any temperature T there exists a $D_I^{**}(T)$ such that there are no equilibrium modes for $D > D_I^{**}(T)$. The single-parameter family of function $D_I^{**}(T)$ (the parameter I) is displayed in Fig. 4.

Therefore, in the case of "low" currents there is an optimal temperature range $[T_1(I), T_2(I)]$ in which equilibrium modes exist for any values of D; conversely, in the case of "high currents a gap width D_I^{**} exists for any temperature T such that equilibrium is impossible for $D \geqslant D^{**}$.

3. To illustrate the results obtained we examine the fluid MÉ-22 (magnetite in ethanol).

A typical value of the volume concentration is $n = 10^{23}\ m^{-3}$. From [2] $m_0 = 0.29 \cdot 10^{-17}$ A/m and $\theta = 858°K$. We find for γ and T_c [4]: $\gamma = 2.95 \cdot 10^{-2}\ N/m \cdot °K^{1.23}$, $T_c = 516.3°K$. The conductor radius is $r_2 = 10^{-2}$ m. We hence obtain for the values of Q and C from (8), (9): $Q = 1.67$ and $C = 3.1\ A^{-2}$. The dimensionless temperature $\overline{T}_{min}$ is $\overline{T}_{min} = 0.547$ according to (12), or in dimensional form $T_{min} = 10°C$.

Let the wetting angle be $\alpha = 60°$. Then the boundary of the "low" and "high" currents domains is found from (11): $\tilde{I} = 0.37$ A.

Therefore, for the current value I = 0.3 A, the optimal temperature range is the range $0.28 < T < 0.77$.

Dependences for several values of I are presented in Figs. 3 and 4 for the fluid MÉ-22.

In conclusion, we note that, as has been noted in [1], the boundary of the domains for equilibrium mode existence in this problem agrees with the boundary of their stability domain relative to axisymmetric disturbances.

NOTATION

r_1, r_2, inner and outer cylinder radii; I, current; T, temperature; χ, magnetic susceptibility; μ_0, a magnetic constant; k_B, Boltzmann constant; n, volume concentration of ferromagnet particles; V, volume of ferromagnet particles; $I_s(T)$, spontaneous magnetization of the particle substance; θ, Curie temperature; σ, is the surface tension coefficient; Bo_m, magnetic Bond number; α_1, α_2, wetting angles on the walls of the outer and inner cylinders, respectively; α_V, coefficient of volume expansion of the solid-phase particles; V_0, particle volume at $T = 0°K$.

LITERATURE CITED

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